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Optimization of Time-Varying Parking Charges and Parking Supply in Networks with Multiple User Classes and Multiple Parking Facilities*

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Abstract: The optimization of parking charges and parking supply over the time of a day is an important problem in the design of transportation networks. This paper presents a bilevel model to determine the optimal time-varying parking charges and parking supply in road networks with multiple user classes and different types of parking facilities. The upper level of the model aims to maximize the network net benefit in response to the parking charges and parking supply, whereas the lower level is a time-dependent network equilibrium problem with elastic demand. A descent-gradient-based solution algorithm is adapted to solve the model. The numerical results show that the implementation of time-varying parking charges and parking supply is useful to effectively cater to the time-varying demand with different parking needs. The model provides a powerful tool for strategically designing parking locations and evaluating various parking policies.

Key words: optimization; time varying; parking charge; parking supply; bilevel model

Introduction

In recent years, parking has become an increasingly serious problem in most densely populated cities around the world. This is mainly attributed to the imbalance between parking demand and supply[1,2]. A shortage of supply increases the searching time for parking and exacerbates traffic congestion on the roads. It also causes more illegal roadside parking.

In response to the growing parking problems in urban areas, it is necessary to optimize the various forms of parking supply in a network, such as provision of on-street, off-street, free, metered, and limited time facilities. This type of optimization belongs to the well-known network design problem[3-5]. In addition, parking charges are one of the most efficient means of traffic demand management[6,7]. Charging may change the choices of travelers of parking location and duration, and the choices of path and departure time as well. Moreover, parking charging schemes are also easier to enforce than road pricing[8-10]. Thus, joint optimization of parking supply and parking charges is of interest.

Shoup[11] argued that minimum parking requirements increase development costs excessively, and that a tax provision can allow developers to reduce these costs. However, the externality cost that is caused by parking was not considered in Shoup's study. Feitelson and Rotem[12] proposed a flat surface parking tax

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approach to internalize the externalities that are associated with parking provision, which is a more effective approach than either the minimal or the maximal parking requirement methods. Anderson and de Palma[13] employed the theory of economics to examine the benefits of parking pricing. Lam et al.[14] proposed a bilevel program model to optimize parking charges, which considers the responses of all travelers to charging.

However, all of these studies were static, and thus cannot reveal the responses of travelers to parking charges and supply over the time of a day or capture the temporal and spatial interaction between supply and demand. Therefore, time-dependent or dynamic versions of these models are worth further study[15-17]. Lam et al.[18] recently proposed a time-dependent network equilibrium model for investigating travel and parking behavior in a network with multiple user classes and multiple parking options. However, in their model the parking charging scheme is pre-specified and the travel demand is fixed and given exogenously. In fact, both the parking supply and charge have significant impacts on travel patterns[7,14,19]. It is, therefore, meaningful to incorporate demand elasticity into parking modeling to capture the responses of travelers to traffic congestion and parking space availability[14].

This paper aims to develop a model that can simultaneously optimize parking charges and parking supply over the time of a day in a network with multiple user classes and various parking facilities. Three types of parking that are popular in densely populated cities are considered[15,19-21]: on-street free-of-charge parking spaces with limited duration, on-street metered parking spaces with limited duration, and off-street parking spaces. The parking supply at the on-street free-of-charge parking locations is assumed to be time-varying and dependent on road congestion. In general, as the traffic demand increases and the level of road congestion rises, the number of permitted on-street free-of-charge parking spaces will be reduced to cater to the increasing road space demand. In contrast, parking charges at on-street metered parking locations and off-street parking locations are time-varying according to the demand for parking spaces, the arrival times of vehicles, and the parking duration. The demand elasticity for parking supply and parking charges is also considered in the model.

1 Basic Considerations

1.1 Travel disutility

Consider a transportation network $G = (N, A)$, where N is the set of all nodes, including origin nodes, intersection nodes, destination nodes, and connector nodes, and A is the set of all directed links, including centroid connectors from origin to road network, road links, parking access links from the connector node to the parking location, parking links and walk links from parking location to the final destination. Let R denote the set of origin nodes, r a single origin node, $r \in R \subset N$, S the set of destination nodes, and s a destination node, $s \in S \subset N$. Let J denote the set of trip purposes of all travelers and j an element in J. Let L denote the set of all possible parking durations (for instance, 1 h, 2 h, 3 h) and l an element in L. Let M denote the set of all user classes and m a user class with a specific trip purpose j and parking duration l. Hence, $M = \{m : (j,l) \in J \times L\}$, where "×" denotes the Cartesian product. Let I denote the set of all parking locations in the network, $I = I_1 \cup I_2 \cup I_3$, where I_1 is the set of all on-street free-of-charge parking locations with limited duration, I_2 is the set of all on-street metered parking locations with limited duration, and I_3 is the set of all off-street parking locations. Denote I_{rs}^m as the set of all feasible parking locations for the origin-destination (OD) pair (r, s) and the user class m.

Suppose that the whole study period $[0, T]$ is discretized into equal time intervals, sequentially numbered $t \in T = \{1, \cdots, \bar{T}\}$, and δ is the length of an interval so that $\bar{T}\delta = T$. The value of T is sufficiently large to ensure that all travelers can complete their journeys within the study period $[0, T]$. Define $C_{rs,i}^m(t)$ as the disutility (measured in time units) for user class m departing from origin r at interval t and traveling to destination s via parking location i, which is the sum of the travel time from origin to parking location, the searching time delay for parking, the parking charge, the walking time from parking location to destination, and the schedule delay cost of early or late arrival at the destination, i.e.

$$C_{rs,i}^m(t) = T_{ri}^m(t) + C_{is}^m\left(t + T_{ri}^m(t)\right) =$$

$$T_{ri}^{m}(t)+\alpha_1^m d_i^m\left(t+T_{ri}^m(t)\right)+\ \alpha_2^m z_i^m\left(t+T_{ri}^m(t)\right)+$$
$$\alpha_3^m w_{is}^m+\alpha_s^m \Theta_s^m\left(t+T_{ri}^m(t)+d_i^m\left(t+T_{ri}^m(t)\right)+w_{is}^m\right),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (1)$$

where $T_{ri}^m(t)$ is the in-vehicle travel time for user class *m* departing from origin *r* at interval *t* and traveling to parking location *i*, $C_{is}^m\left(t+T_{ri}^m(t)\right)$ is the disutility from parking location *i* to destination *s*, $d_i^m\left(t+T_{ri}^m(t)\right)$ is the searching time delay for an available parking space for user class *m* departing from origin *r* at interval *t* and arriving at location *i* at interval $t+T_{ri}^m(t)$, $z_i^m\left(t+T_{ri}^m(t)\right)$ is the parking charge for user class *m* at location *i* at interval $t+T_{ri}^m(t)$, w_{is}^m is the walking time of user class *m* from location *i* to destination *s*, and $\Theta_s^m\left(t+T_{ri}^m(t)+d_i^m\left(t+T_{ri}^m(t)\right)+w_{is}^m\right)$ is the schedule delay cost of early or late arrival at destination *s* for user class *m* departing from origin *r* at interval *t* and arriving at destination *s* via parking location *i* at interval $t+T_{ri}^m(t)+d_i^m\left(t+T_{ri}^m(t)\right)+w_{is}^m$. Some terms in Eq. (1) are multiplied by coefficients (α) for the purpose of converting different quantities to the same unit defined in the disutility function.

Suppose that the searching time delay function for an available parking space follows the form of the Bureau of Public Roads (BPR) type function[14],

$$d_i^m(t+T_{ri}^m(t))=d_i^{0m}+0.15\left(D_i\left(t+T_{ri}^m(t)\right)/h_i\right)^4,$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (2)$$

where d_i^{0m} is the free-flow parking access time at parking location *i*, $D_i\left(t+T_{ri}^m(t)\right)$ is the accumulation or occupancy of parking location *i* when the travelers in user class *m* depart at interval *t* from origin *r* and arrive at the location *i* at interval $t+T_{ri}^m(t)$, and h_i is the capacity of parking location *i*. For the on-street free-of-charge parking spaces with limited duration, the parking capacity h_i is a function of the time *t* of a day, i.e., $h_i=h_i(t)$.

The accumulation at parking location *i* can be represented in terms of the path inflow only[18]. For the OD pair (*r*, *s*), the cumulative arrivals of user class *m*, $U_{ri}^m(t)$, at parking location *i* by interval *t* (but excluding interval *t*) are given by

$$U_{ri}^m(t)=\sum_{\xi=1}^{t-1}\ \sum_{k:k+T_{ri}^m(k)=\xi}\ \sum_{p\in P_{ri}} f_{ri,p}^m(k),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (3)$$

where P_{ri} is the set of all paths between origin *r* and parking location *i*, and $f_{ri,p}^m(k)$ is the departure flow rate of user class *m* on path *p* between origin *r* and parking location *i* during interval *k*. Then, the total cumulative arrivals of user class *m* from all of the OD pairs, $U_i^m(t)$, at parking location *i* by interval *t* can be computed by

$$U_i^m(t)=\sum_{r,s}U_{ri}^m(t),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (4)$$

On the other hand, for a given OD pair and user class *m* with parking duration *l*, travelers who arrive at parking location *i* before interval (*t*–*l*) have already left location *i* before interval *t*, and thus the cumulative departures of user class *m*, $V_{ri}^m(t)$, from location *i* by interval *t* are given by

$$V_{ri}^m(t)=\sum_{\xi=1}^{t-1-l}\ \sum_{k:k+T_{ri}^m(k)=\xi}\ \sum_{p\in P_{ri}} f_{ri,p}^m(k),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (5)$$

Therefore, the total cumulative departures of user class *m* for all of the OD pairs, $V_i^m(t)$, from parking location *i* by interval *t* can be computed by

$$V_i^m(t)=\sum_{r,s}V_{ri}^m(t),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (6)$$

The parking accumulation at location *i* by user class *m* at interval *t*, $D_i^m(t)$, which equals to the cumulative arrivals, $U_i^m(t)$, by the time *t* minus the cumulative departures, $V_{ri}^m(t)$, by that time, can be represented as

$$D_i^m(t)=U_i^m(t)-V_i^m(t),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (7)$$

Hence, the total parking accumulation at location *i* by all user classes at interval *t*, $D_i(t)$, is

$$D_i(t)=\sum_m D_i^m(t),\ \ \forall i\in I,\ t\in T \qquad (8)$$

The parking charge, which depends on the desired parking duration and the arrival time interval at the parking location, can be defined as[2,18]

$$z_i^m\left(t+T_{ri}^m(t)\right)=\theta_m\times l\times p_i\left(t+T_{ri}^m(t)\right),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (9)$$

where θ_m is the parking charge discount for user class *m*, and $p_i\left(t+T_{ri}^m(t)\right)$ is the hourly parking fee at time interval $t+T_{ri}^m(t)$. In general, the longer the parking duration, the smaller the value of θ_m.

The walking time, which depends on the distance from the parking location to the users' final destination and walking speed[22], can be defined as

$$w_{is}^m=\Gamma(i,s)/\omega,\ \forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S \quad (10)$$

where $\Gamma(i,s)$ is the average walking distance from parking location *i* to final destination *s*, and ω is average walking speed of travelers (km/h).

As shown in Refs. [23, 24], the schedule delay cost of arrival at the destination *s* can be defined as

$$\Theta_s^m\left(\tilde{t}(t)\right)=\begin{cases}\tau^m\left(t_s^{m*}-\Delta_s^m-\tilde{t}(t)\right), & \text{if } t_s^{m*}-\Delta_s^m>\tilde{t}(t),\\ \lambda^m\left(\tilde{t}(t)-t_s^{m*}-\Delta_s^m\right), & \text{if } t_s^{m*}+\Delta_s^m<\tilde{t}(t),\\ 0, & \text{otherwise}\end{cases} \qquad (11)$$

where the arrival time $\tilde{t}(t)=t+T_{ri}^m(t)+d_i^m(t+T_{ri}^m(t))+w_{is}^m$, $[t_s^{m*}-\Delta_s^m, t_s^{m*}+\Delta_s^m]$ is the window of the desired arrival time of user class *m* at destination *s* without any schedule delay penalty, t_s^{m*} is the middle point of the time window, and $\tau^m(\lambda^m)$ is the unit cost of early (late) schedule delay at destination *s* for user class *m*.

1.2 Behavior modeling

Before making a trip, a traveler first determines the start and end times of the activity, namely, the departure time and parking duration, and then selects a desirable parking location that will minimize the travel disutility from origin to final destination. After choosing the parking location, the traveler will take the shortest route to travel to the parking location. In this paper, we make use of the multinomial logit model to describe the hierarchical choice process of travelers. Let q_{rs}^j be the total demand of travelers with trip purpose *j* between OD pair (*r*, *s*), and $q_{rs}^{jl}(t)$ (or $q_{rs}^m(t)$) be the portion of the demand that selects interval *t* for departure and parks their cars for *l* intervals. We then have

$$q_{rs}^m(t)=q_{rs}^j\frac{\exp\left(-\beta_m C_{rs}^{m(j,l)}(t)\right)}{\sum_{t\in T}\sum_{l\in L}\exp\left(-\beta_m C_{rs}^{m(j,l)}(t)\right)},$$
$$\forall m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (12)$$

where $C_{rs}^{m(j,l)}(t)$ is the travel disutility of user class *m* departing at interval *t* between OD pair (*r*, *s*), and β_m is a dispersion parameter that reflects the degree of familiarity of travelers with, or their perception of, the variation of travel disutility.

Now consider the response of the travel demand elasticity to the road congestion and the availability of parking supply. Following Yang and Meng[24] and Szeto and Lo[25], the total demand q_{rs}^j is assumed to be a decreasing function of the log-sum travel disutility, S_{rs}^j, between that OD pair, i.e.,

$$q_{rs}^j=D_{rs}^j(S_{rs}^j),\ \forall j\in J,\ r\in R,\ s\in S \qquad (13)$$

$$S_{rs}^j=-\frac{1}{\beta_m}\ln\sum_{l,t}\exp\left(-\beta_m C_{rs}^{m(j,l)}(t)\right),$$
$$\forall j\in J,\ r\in R,\ s\in S \qquad (14)$$

Following Lam et al.[18], a deterministic parking location choice equilibrium can mathematically be expressed in a complementary form as

$$\left(C_{rs,i}^m(t)-C_{rs}^m(t)\right)q_{rs,i}^m(t)=0,$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (15)$$

$$C_{rs,i}^m(t)\geqslant C_{rs}^m(t),$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (16)$$

$$q_{rs,i}^m(t)\geqslant 0,$$
$$\forall i\in I_{rs}^m,\ m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (17)$$

$$C_{rs}^m(t)=\min_{i\in I_{rs}^m}C_{rs,i}^m(t),$$
$$\forall m\in M,\ r\in R,\ s\in S,\ t\in T \qquad (18)$$

where $q_{rs,i}^m(t)$ is the travel demand of user class *m* between OD pair (*r*, *s*) departing at interval *t* via parking location *i*.

In the following, we describe mathematically a deterministic time-dependent multi-class route choice equilibrium problem, which implies that for each OD pair and each user class at each interval, the actual path travel times experienced by travelers departing at the same time should be equal and minimal[23,25-27]. Let $T_{ri,p}^m(t)$ be the path travel time of user class *m* departing from origin *r* at interval *t* to parking location *i* via path p. The time-dependent route choice equilibrium can be mathematically written as

$$\left(T_{ri,p}^{m}(t)-T_{ri}^{m}(t)\right) f_{ri,p}^{m}(t)=0,$$

$$\forall p\in P_{ri},\ i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (19)$$

$$T_{ri,p}^{m}(t)\geqslant T_{ri}^{m}(t),$$

$$\forall p\in P_{ri},\ i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (20)$$

$$f_{ri,p}^{m}(t)\geqslant 0,$$

$$\forall p\in P_{ri},\ i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (21)$$

$$T_{ri}^{m}(t)=\min_{p\in P_{ri}} T_{ri,p}^{m}(t),$$

$$\forall i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (22)$$

$$T_{ri,p}^{m}(t)=\sum_{a}\sum_{k\geqslant t} c_{a}^{m}(k)\delta_{apt}^{rim}(k),$$

$$\forall p\in P_{ri},\ i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (23)$$

where $\delta_{apt}^{rim}(k)$, an indicator variable, is equal to 1 if the flow on path p departing from origin r at interval t to parking location i arrives at link a at interval k, and 0 otherwise. $c_{a}^{m}(k)$ is the travel time of user class m on link a during interval k, which can be represented as a function of the inflows of all the user classes entering that link by interval k[27],

$$c_{a}^{m}(k)=f\left(u_{a}^{1}(1),u_{a}^{1}(2),\ldots,u_{a}^{1}(k);\ldots;\right.$$

$$\left.u_{a}^{m}(1),u_{a}^{m}(2),\ldots,u_{a}^{m}(k)\right),$$

$$\forall a\in A,\ m\in M,\ k\in T \quad (24)$$

$$u_{a}^{m}(k)=\sum_{ri}\sum_{p}\sum_{t} u_{apt}^{rim}(k),\ \ \forall a\in A,\ m\in M,\ k\in T \quad (25)$$

where $u_{apt}^{rim}(k)$ is the inflow to link a during time interval k that departs from origin r over path p toward parking location i during interval t. The link inflow can be represented by the path inflow through the use of the indicator variable $\delta_{apt}^{rim}(k)$, i.e.,

$$u_{apt}^{rim}(k)=f_{ri,p}^{m}(t)\delta_{apt}^{rim}(k),\ \ \forall a\in A,\ p\in P_{ri},$$

$$i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T,\ k\in T \quad (26)$$

Up to now, we have defined and formulated all the travel choices that are to be investigated in this paper. The parking location and route choices follow a time-dependent deterministic user equilibrium. The combination of departure time and parking duration choices is governed by a multinomial logit formulation. Hence, a flow pattern $\left\{f_{ri,p}^{m*}(t),q_{rs,i}^{m*}(t),q_{rs}^{m*}(t),q_{rs}^{j*}\right\}$ is a time-dependent network equilibrium for the joint choice problem of departure time, parking duration, parking location, and travel route in a network with elastic demand if it satisfies conditions (12)-(23) simultaneously.

2 Model Formulation

The government's objective is to maximize the system's net benefit and all individuals then adjust their travel behavior according to the supply and charges given by the government. The interaction between the government and individuals can be represented by a bilevel formulation. The lower level of the model is a time-dependent network equilibrium problem with elastic demand, which can be formulated as an equivalent variational inequality (VI) formulation as follows[18].

$$\sum_{ri}\sum_{m}\sum_{p}\sum_{t} T_{ri,p}^{m*}(t)\left(f_{ri,p}^{m}(t)-f_{ri,p}^{m*}(t)\right)+$$

$$\sum_{rs}\sum_{m}\sum_{i}\sum_{t} C_{is}^{m*}\left(t+T_{ri}^{m*}(t)\right)\left(q_{rs,i}^{m}(t)-q_{rs,i}^{m*}(t)\right)+$$

$$\sum_{rs}\sum_{m}\sum_{t}\frac{1}{\beta_{m}}\ln q_{rs}^{m*}(t)\left(q_{rs}^{m}(t)-q_{rs}^{m*}(t)\right)+$$

$$\sum_{rs}\sum_{j} D_{rs}^{j^{-1}}(q_{rs}^{j*})\left(q_{rs}^{j}-q_{rs}^{j*}\right)\geqslant 0 \quad (27)$$

subject to

$$\sum_{p} f_{ri,p}^{m}(t)=q_{rs,i}^{m}(t),$$

$$\forall i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (28)$$

$$\sum_{i} q_{rs,i}^{m}(t)=q_{rs}^{m}(t),\ \ \forall m\in M,\ r\in R,\ s\in S,\ t\in T \quad (29)$$

$$\sum_{l,t} q_{rs}^{m(j,l)}(t)=q_{rs}^{j},\ \ \forall j\in J,\ r\in R,\ s\in S \quad (30)$$

$$f_{ri,p}^{m}(t)\geqslant 0,$$

$$\forall p\in P_{ri},\ i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (31)$$

$$q_{rs,i}^{m}(t),q_{rs}^{m}(t),q_{rs}^{j}\geqslant 0,$$

$$\forall i\in I_{rs}^{m},\ m\in M,\ r\in R,\ s\in S,\ t\in T \quad (32)$$

where Eq. (28) states the conservation of path flows, i.e., the sum of all the path flows of user class m between origin r and parking location i should equal the travel demand between OD pair (r, s) when parking location i is selected. Equation (29) is the parking demand conservation constraint. Equation (30) shows that the sum of demand over all the intervals and parking durations for each OD pair should equal the total OD demand with trip purpose j between that OD pair. In addition, the following definitional constraints should be included:

$$T_{ri,p}^{m}(t)=\sum_{a}\sum_{k\geqslant t}c_{a}^{m}(k)\delta_{apt}^{rim}(k),$$

$$\forall p\in P_{ri},\ \ i\in I_{rs}^{m},\ \ m\in M,\ \ r\in R,\ \ s\in S,\ \ t\in T \quad (33)$$

$$u_{a}^{m}(k)=\sum_{ri}\sum_{p}\sum_{t}f_{ri,p}^{m}(t)\delta_{apt}^{rim}(k),$$

$$\forall a\in A,\ \ m\in M,\ \ k\in T \quad (34)$$

In the upper level, the government's objective is to maximize the network net benefit under the road and parking supply constraints for acceptable ranges of the parking supply and parking charges, i.e.,

$$\max_{\boldsymbol{h},\boldsymbol{p}} J=\sum_{rs}\sum_{j}\int_{0}^{q_{rs}^{j*}}D_{rs}^{j^{-1}}(w)\mathrm{d}w-$$

$$\sum_{rs}\sum_{m}\sum_{i}\sum_{t}q_{rs,i}^{m*}(t)C_{rs,i}^{m}(\boldsymbol{h},\boldsymbol{p})-$$

$$\rho_1\sum_{t}\sum_{i\in I_1}h_i(t)-T(\rho_2\sum_{i\in I_2}h_i+\rho_3\sum_{i\in I_3}h_i) \quad (35)$$

subject to

$$h_i^{\min}\leqslant h_i(t)\leqslant h_i^{\max},\ \ \forall i\in I_1,\ \ t\in T \quad (36)$$

$$h_i^{\min}\leqslant h_i\leqslant h_i^{\max},\ \ \forall i\in I_2\cup I_3 \quad (37)$$

$$p_i^{\min}\leqslant p_i(t)\leqslant p_i^{\max},\ \ \forall i\in I_2\cup I_3,\ \ t\in T \quad (38)$$

$$u_a(\boldsymbol{h},\boldsymbol{p})\leqslant C_a,\ \ \forall a\in A \quad (39)$$

$$D_i(\boldsymbol{h},\boldsymbol{p})\leqslant h_i(t),\ \ \forall i\in I_1,\ \ t\in T \quad (40)$$

$$D_i(\boldsymbol{h},\boldsymbol{p})\leqslant h_i,\ \ \forall i\in I_2\cup I_3,\ \ t\in T \quad (41)$$

where $\boldsymbol{h}=\left(h_i,h_i(t)\right)^{\mathrm{T}}$, $\boldsymbol{p}=\left(p_i,p_i(t)\right)^{\mathrm{T}}$, $\forall i\in I,t\in T$ are the vectors of the parking supply and parking charges, respectively. $h_i^{\max}$ ($h_i^{\min}$) and $p_i^{\max}$ ($p_i^{\min}$) are the upper (lower) bounds of the parking supply and parking charges at parking location *i*, respectively. ρ_1, ρ_2, and ρ_3 are the scaling factors converting construction and operation costs to travel costs. The equilibrium travel demand (q_{rs}^{j*}, $q_{rs,i}^{m*}(t)$) is obtained by solving the lower-level time-dependent network equilibrium problem shown in Eqs. (27)-(34). The first term in expression (35) is the total user benefit from travel, the second term in the expression is the total social cost incurred by all users, and the last two terms are the equivalent travel costs for constructing and operating parking facilities. Equations (36) and (37) are the feasible bounds of parking supply at the three parking facilities, respectively. Equation (38) is the acceptable range of parking charges at the on-street metered with limited duration and the off-street parking spaces. Equations (39)-(41) are the road link and parking capacity constraints, respectively.

3 Solution Algorithm

There is a body of solution algorithms for solving the bilevel programming problem[28-30]. In this paper, a descent-gradient-based strategy is adapted to search for the local optimum of the bilevel model. The upper-level problem, i.e., the objective function together with the road link and parking capacity constraints, consists of nonlinear and implicit functions with decision variables $\boldsymbol{h}$ and $\boldsymbol{p}$. Thus, local linear approximations using Taylor's formula can be implemented on the basis of the derivatives of travel demand, link inflows, and parking accumulations with respect to the parking supply and parking charges:

$$q_{rs}^{j}(\boldsymbol{h},\boldsymbol{p})\approx q_{rs}^{j}(\boldsymbol{h}^*,\boldsymbol{p}^*)+\nabla_{\boldsymbol{h}}q_{rs}^{j}(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{h}-\boldsymbol{h}^*)+$$
$$\nabla_{\boldsymbol{p}}q_{rs}^{j}(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{p}-\boldsymbol{p}^*) \quad (42)$$

$$q_{rs,i}^{m}(\boldsymbol{h},\boldsymbol{p})\approx q_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)+\nabla_{\boldsymbol{h}}q_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{h}-\boldsymbol{h}^*)+$$
$$\nabla_{\boldsymbol{p}}q_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{p}-\boldsymbol{p}^*) \quad (43)$$

$$C_{rs,i}^{m}(\boldsymbol{h},\boldsymbol{p})\approx C_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)+\nabla_{\boldsymbol{h}}C_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{h}-\boldsymbol{h}^*)+$$
$$\nabla_{\boldsymbol{p}}C_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{p}-\boldsymbol{p}^*) \quad (44)$$

$$u_a(\boldsymbol{h},\boldsymbol{p})\approx u_a(\boldsymbol{h}^*,\boldsymbol{p}^*)+\nabla_{\boldsymbol{h}}u_a(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{h}-\boldsymbol{h}^*)+$$
$$\nabla_{\boldsymbol{p}}u_a(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{p}-\boldsymbol{p}^*) \quad (45)$$

$$D_i(\boldsymbol{h},\boldsymbol{p})\approx D_i(\boldsymbol{h}^*,\boldsymbol{p}^*)+\nabla_{\boldsymbol{h}}D_i(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{h}-\boldsymbol{h}^*)+$$
$$\nabla_{\boldsymbol{p}}D_i(\boldsymbol{h}^*,\boldsymbol{p}^*)(\boldsymbol{p}-\boldsymbol{p}^*) \quad (46)$$

where $\nabla_{\boldsymbol{h}}q_{rs}^{j}$, $\nabla_{\boldsymbol{p}}q_{rs}^{j}$, $\nabla_{\boldsymbol{h}}q_{rs,i}^{m}$ and $\nabla_{\boldsymbol{p}}q_{rs,i}^{m}$, can be obtained by the method of sensitivity analysis for the network equilibrium problem[31,32]. $\nabla_{\boldsymbol{h}}C_{rs,i}^{m}$, $\nabla_{\boldsymbol{p}}C_{rs,i}^{m}$, $\nabla_{\boldsymbol{h}}u_a$, $\nabla_{\boldsymbol{p}}u_a$, $\nabla_{\boldsymbol{h}}D_i$, and $\nabla_{\boldsymbol{p}}D_i$ can be obtained by adopting the method that was proposed in Refs. [33-35]. $\boldsymbol{h}^*$ and $\boldsymbol{p}^*$ represent the solutions at the current iteration, $q_{rs}^{j}(\boldsymbol{h}^*,\boldsymbol{p}^*)$, $q_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)$, $C_{rs,i}^{m}(\boldsymbol{h}^*,\boldsymbol{p}^*)$, $u_a(\boldsymbol{h}^*,\boldsymbol{p}^*)$, and $D_i(\boldsymbol{h}^*,\boldsymbol{p}^*)$ are the corresponding values at equilibrium state. Equations (45) and (46) are then applied to Eqs. (39)-(41) to form a set of linear constraints, and Eqs. (42)-(44) are substituted into the objective function (35). Consequently, the upper-level problem is converted to a standard quadratic programming problem which can be solved using existing solution algorithms such as the gradient projection method[36].

The step-by-step procedure of the solution algorithm is given below.

Step 1 Determine the initial parking supply $\boldsymbol{h}^{(k)}$ and parking charge $\boldsymbol{p}^{(k)}$. Set $k = 0$.

Step 2 Solve the lower-level network equilibrium problem for the given $\boldsymbol{h}^{(k)}$ and $\boldsymbol{p}^{(k)}$, and denote the solution as q_{rs}^{j*} and $q_{rs,i}^{m*}(t)$.

Step 3 Use the sensitivity analysis method to calculate the derivatives $\nabla_{\boldsymbol{h}} q_{rs}^j$, $\nabla_{\boldsymbol{p}} q_{rs}^j$, $\nabla_{\boldsymbol{h}} q_{rs,i}^m$, $\nabla_{\boldsymbol{p}} q_{rs,i}^m$, $\nabla_{\boldsymbol{h}} C_{rs,i}^m$, $\nabla_{\boldsymbol{p}} C_{rs,i}^m$, $\nabla_{\boldsymbol{h}} u_a$, $\nabla_{\boldsymbol{p}} u_a$, $\nabla_{\boldsymbol{h}} D_i$, and $\nabla_{\boldsymbol{p}} D_i$, and then obtain the gradients of the objective function of the upper level of the model with respect to parking supply and parking charge, that is, $\nabla_{\boldsymbol{h}} J$ and $\nabla_{\boldsymbol{p}} J$.

Step 4 Use the derivative information to construct an approximate quadratic program for the upper-level problem, solve this quadratic program, and obtain the new parking supply $\boldsymbol{h}^{(k+1)}$ and parking charge $\boldsymbol{p}^{(k+1)}$.

Step 5 If the pre-specified stopping criterion is satisfied, then stop and output the solution; otherwise, set $k = k+1$ and go to Step 2.

In Step 2, the lower-level time-dependent network equilibrium problem can be solved by the decomposition algorithm proposed by Lam et al.[18]

4 A Numerical Experiment

4.1 Experiment settings

The test network, as shown in Fig. 1, consists of four OD pairs (1-3, 1-4, 2-3, and 2-4), six road nodes, eleven road links, and six parking locations (two off-street parking locations P1 and P2, two on-street metered parking locations with limited duration A and B, and two on-street free-of-charge parking locations with limited duration C and D). The analysis assumes that the permitted maximum parking durations for all on-street parking locations A, B, C, and D are identical at three hours. The study period is from 06:00 to 20:00, and is divided into 14 hourly intervals. The travelers are grouped into two user classes according to whether they are commuters or non-commuters. The non-commuters are further categorized into three subclasses according to their parking durations of 1 h, 2 h, or 3 h, and the commuters are partitioned into two subclasses according to their parking durations of 4 h or 8 h.

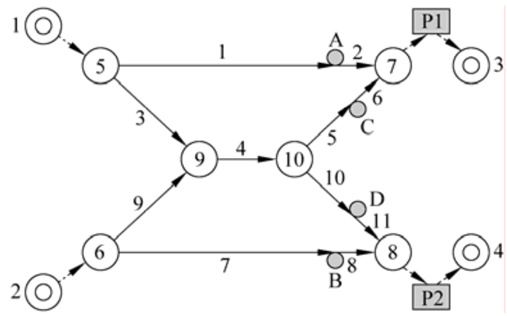

Fig. 1 Test network

The adopted time-dependent link travel time function is simply formulated as a BPR-type,

$$c_a(t) = c_a^0\left(1.0 + 0.15 \times \left(u_a(t)/C_a\right)^4\right),$$
$$\forall a \in A,\ \ t \in T \tag{47}$$

where c_a^0 is the free-flow travel time, C_a is the capacity of link a, and $u_a(t) = \sum_m u_a^m(t)$. The parameters of all the link travel time functions are given in Table 1.

Table 1 Input parameters associated with the link travel time function

Link No.	c_a^0 / h	C_a /(veh • h^{-1})
1 and 7	0.60	1000
2 and 8	0.10	500
3 and 9	0.30	1000
4	0.30	800
5 and 10	0.20	1000
6 and 11	0.10	500

The exponential demand function is adopted and specified as

$$q_{rs}^j = D_{rs}^j(S_{rs}^j) = \bar{q}_{rs}^j \exp(-\pi_{rs}^j S_{rs}^j),$$
$$\forall j \in J,\ \ r \in R,\ \ s \in S \tag{48}$$

where π_{rs}^j is a positive parameter and $\bar{q}_{rs}^j$ is the potential demand with trip purpose j between OD pair (r, s). The parameters of the demand function are shown in Table 2.

The input parameters associated with the travel disutility function and the parking characteristics in the model are shown in Tables 3 and 4. Suppose that the average walking speed in Eq. (10) is $\omega = 5.0$ km/h. The average walking distances from P1 to Destination 3 and from P2 to Destination 4 are both 0.75 km. The

Table 2 Input parameters associated with the demand function

	$\bar{q}_{13}$	$\bar{q}_{14}$	$\bar{q}_{23}$	$\bar{q}_{24}$	π_{13}	π_{14}	π_{23}	π_{24}	β
Commuter	1500	1000	1000	1500	0.03	0.03	0.03	0.03	0.45
Non-commuter	800	500	500	800	0.05	0.05	0.05	0.05	0.15

Table 3 Input parameters associated with the travel disutility function

	α_1	α_2	α_3	α_s	τ	λ	$[t^* + \Delta,\ t^* - \Delta]$
Commuter	1.4	0.1	1.8	0.3	6.0	25.0	[8:45, 9:15]
Non-commuter	1.4	0.2	2.0	0.3	4.0	15.0	[12:00, 14:00]

Table 4 Input parameters associated with the parking characteristics

Parking facility	Free-flow parking access time (h)	Bounds of parking supply	Bounds of parking charges (HK$)
P1 and P2	0.15	[900, 1500]	[12, 18]
A and B	0.10	[100, 300]	[6, 12]
C and D	0.10	[150, 250]	——

average walking distances from all on-street parking locations A, B, C, and D to the nearby destinations are 0.25 km. The discount coefficients θ_m in Eq. (9) for all user classes are assumed to be 1.0. The scaling factors in expression (35) are $\rho_1 = 0.25$, $\rho_2 = 0.5$, and $\rho_3 = 1.0$.

The symmetric network structure and input data imply that some OD pairs have the same traffic demand and some paths and links will experience the same traffic conditions such as inflow rates and travel costs. This symmetry is not specially designed to simplify the problem but to facilitate the presentation of the numerical results. The symmetric outputs were found in the results, even though the computer program treated all OD pairs, paths, and links independently.

4.2 Analysis of numerical results

First analyze the numerical results in the context of the time-varying parking charges and parking supply. The realized travel demands are $q_{13}^{\mathrm{c}} = q_{24}^{\mathrm{c}} = 1328.74$ and $q_{14}^{\mathrm{c}} = q_{23}^{\mathrm{c}} = 869.23$ for commuters, and $q_{13}^{\mathrm{nc}} = q_{24}^{\mathrm{nc}} =$ 653.19 and $\bar{q}_{14}^{\mathrm{nc}} = \bar{q}_{23}^{\mathrm{nc}} = 405.37$ for non-commuters. The optimal parking supplies at on-street metered parking location A (or B) and off-street parking location P1 (or P2) are $h_{\mathrm{A}} = h_{\mathrm{B}} = 208.16$ and $h_{\mathrm{P1}} = h_{\mathrm{P2}} =$ 1127.82.

Figure 2 shows that the highest parking charge occurs between 12:00 and 14:00 for parking location A, and that two charging peaks occur for parking location P1, one during interval 08:00-09:00 and another during 12:00-14:00. This is because these intervals correspond to the traffic rush hours. In general, the commuters tend to depart before 09:00 so that they can arrive at their places of work punctually. The arrival peak for the commuters at parking location P1 occurs between 08:00 and 09:00. In contrast, the non-commuters prefer to depart between 10:00 and 13:00 to avoid a schedule delay penalty, and park their cars at A and P1 when they arrive between 12:00 and 14:00. After 14:00, the parking charges are reduced because the level of traffic congestion declines.

Figure 3 shows that the number of required parking spaces first decreases with the time interval, reaching a minimum during the interval 13:00-14:00 (158.29), and then increases toward the upper bound of supply. Combining Figs. 2 and 3 shows that during the rush hours, few on-street free of charge parking locations are open, and higher parking fees are levied at on-street metered parking locations and off-street parking locations. The reverse occurs during off-peak hours. The objective of this scheme is to guarantee that travelers can arrive at their destinations quickly during peak hours by increasing the link capacity. Providing more on-street parking spaces during off-peak hours serves the purpose of allowing people to find parking locations conveniently. Therefore, the adjustment of parking spaces and parking charges is a powerful measure for managing traffic demand effectively and complementarily.

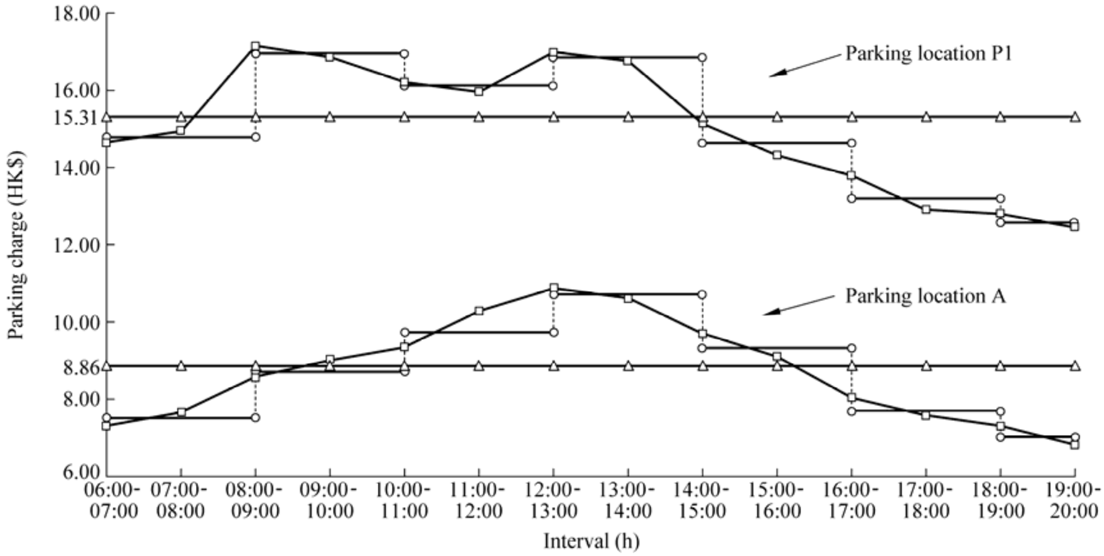

Fig. 2 Optimal charging schemes at on-street metered parking location A and off-street parking location P1 over the study horizon

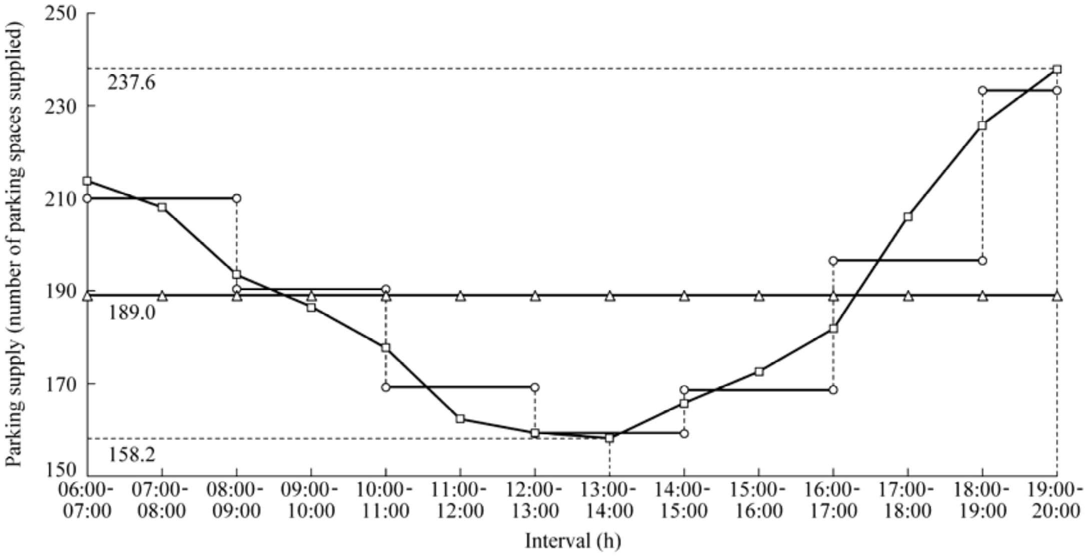

Fig. 3 Optimal supply pattern at on-street free of charge parking location C

Figures 2 and 3 also show the step and uniform charging schemes and supply patterns. We compare them with the optimal time-varying schemes and patterns given by the model in this paper. The network net benefits are 83 921 units by the time-varying pattern, 83 094 units by the step pattern, and 79 825 units by the uniform pattern. Therefore, it is more attractive to set time-varying parking charges and parking supply to cater to the time-varying traffic demand. The step pattern is designed to be closer to the time-varying pattern to create a reasonable efficiency. The step alternative is recommended since it is much easier to implement in practice.

5 Conclusions

This paper describes a bilevel programming model to optimize the number of parking spaces and the parking charges while accounting for road and parking capacity constraints and acceptable bounds on parking supply and charges. The travel demand is elastic, and thus responds to traffic congestion and parking space availability. In this model, the upper level aims to maximize the system's net benefit, while the lower level is a time-dependent network equilibrium problem that simultaneously considers the choices of travelers on departure time, route, parking location, and parking duration. The model was solved by a descent-gradient-based algorithm. The numerical results show that the modeling approach captures the behavioral characteristics of travelers in choosing parking facilities in a dynamic context and acquires the essential economic attributes for the assessment of the operational performance of the parking facilities.

The model provides a versatile and systematic framework that serves as a useful tool for strategically planning and designing parking facilities. The model also helps in understanding the economics of pricing parking in a dynamic paradigm. For example, the model can be further used to investigate the effects of time-varying parking charges on the activities of travelers[37] and on scheduling reliability of morning commuting[38]. This advances the understanding of the interactions between parking demand and supply and opens the way toward a more comprehensive and effective formulation of pricing policy in the transportation system.

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